Reconciling Alternative Views About the Appropriate Social Discount Rate

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This paper shows that in an economy with a capital income tax distortion, an exogenous pre-tax rate of return, and lump sum taxes being feasible, the SOC, MCF and STP criteria all correctly identify worthwhile projects provided the criteria are properly applied. Apparent differences between the SOC and MCF criteria arise from different interpretations of a project's indirect revenue effect. Apparent differences between the SOC and STP criteria arise from different assumptions about the private sector's knowledge of the project's benefits and costs.

1 Introduction

This paper is an attempt to reconcile three prevalent criteria for project evaluation in a tax distorted economy: the social opportunity cost of capital (SOC) criterion proposed by Harberger (1973) and Sandmo-Dreze (1971), which discounts benefits and costs at the rate of return foregone in the private sector (a weighted average of the pre-tax and post-tax rates of return); the social rate of time preference approach (STP) proposed by Marglin (1964), Feldstein (1972), Bradford (1975) and Lind (1982), which converts benefits and costs into their "consumption equivalents" by multiplying all investment displaced (or induced) by the shadow price of capital and discounting at the social rate of time preference (which may or may not equal the after tax rate of return); and the marginal cost of funds (MCF) approach recently proposed by Liu (2003), which discounts benefits at the after-tax rate of return and discounts costs (including any "indirect revenue effects") at the pre-tax rate of return, but multiplies all costs and indirect revenue effects by a parameter referred to as the MCF.

The MCF criterion recognizes that raising an additional dollar of revenue using a lump sum tax will have a social welfare cost of more than a dollar when there is a pre-existing capital income tax distortion. Reasoning from the criterion for the optimum supply of a public good in a tax-distorted economy when benefits and costs are contemporaneous, Liu (2003) argues that the marginal cost of funds must be an integral part of any multi-period project evaluation, and the SOC and STP criteria are both deficient because they fail to take the MCF into account. He further argues that there is no general formula for the weights that are needed to calculate the social opportunity cost of capital, so the appropriate discount rate is "project specific" making the SOC criterion almost impossible to apply in practice. The STP approach suffers from the same project dependence problem as well as controversy over how to measure the shadow price of capital. The MCF criterion supposedly avoids these difficulties because the appropriate discount rate for evaluating benefits and costs and the MCF parameter are all project independent.

However, I believe there is some misunderstanding about how to implement the SOC criterion. If a project produces benefits that the private sector treats as equivalent to income a straightforward application of the SOC criterion is appropriate; benefits and costs should be discounted at the social (economic) opportunity cost of capital, which is a weighted average of the pre-tax and post-tax rates of return where the weights reflect the proportions of funding that displace private investment and consumption respectively when the government borrows to finance the project. For any project whose benefits are not treated as income there will be "indirect revenue effects", but they should be incorporated by adding to (or subtracting from) the project's benefits, not by adjusting the discount rate. No matter what the nature of the benefits, it is not necessary to introduce the MCF parameter when applying the SOC criterion unless the indirect revenue effects that are relevant when applying the MCF criterion are known and the indirect revenue effects that are relevant when applying the SOC criterion are not. When properly implemented, the SOC criterion is perfectly consistent with the MCF criterion; both criteria correctly identify all worthy projects.

Insofar as the STP criterion is concerned, it yields results that are equivalent to the MCF and SOC criteria if the private sector is fully informed about the project's benefits and costs. However, the STP criterion, as it is normally applied, treats the private sector as myopic with respect to the project's benefits and costs even though this information is known by the planner. Even if the STP criterion can be rescued there are serious problems of implementation because a dollar of project expenditure will have a different "consumption equivalent" depending upon when that dollar is spent.

Section 2 demonstrates the fundamental equivalence between the MCF and the SOC criteria by noting that they require different measures of a project's indirect revenue effect. Section 3 shows that the STP criterion yields results that are equivalent to the SOC and MCF criteria if the private sector is as well informed as the planner about the project's benefits and costs. Section 4 concludes by emphasizing the limitations of the MCF and STP criteria and the practical advantages of the SOC criterion.

2 The MCF criterion versus the SOC criterion

Consider the following simplified version of the infinitely lived representative agent (ILA) model used by Liu (2003). The representative agent earns an exogenous pre-tax wage w for a given supply of work effort, and earns an exogenous pre-tax rate of return ρ on assets, but incurs a tax T^t on labour income and a tax at proportional rate τ on capital income.[1] There are two goods available in each period: a composite private good c^t, and a publicly provided good g^t.

Given the time paths of the publicly provided good $\{g^t\}$and taxes $\{T^t\}$, the representative agent chooses a time path for private consumption $\{c^t\}$ to maximize

$\Sigma_{t=0}^{\infty}\beta^t U(c^t, g^t)$

subject to

$\Sigma_{t=0}^{\infty} c^t/(1+r)^t = \Sigma_{t=0}^{\infty}(w - T^t)/(1+r)^t + A^0$

where A^0 is initial assets and β is the pure rate of time preference. The consumer's discount rate is the after tax rate of return $r = (1-\tau)\rho$.

Following Liu (2003), I assume that the benefits of the publicly provided good cannot be appropriated through user fees or normal market processes. The government's budget constraint therefore requires that the discounted sum of tax revenue $\{R^t\}$ minus project expenditures $\{I_g^t\}$ must equal its initial net indebtedness D^0. Tax revenue in period t is lump sum taxes plus capital income taxes, and capital income taxes depend upon assets at the beginning of period t, which depend upon the time stream of lump sum taxes $T = \{T^t\}$

[1] Labour supply is exogenous, so any tax on labour income is effectively a lump sum tax. The model could be generalized by introducing labour-leisure choice as in Liu (2003), but this would just complicate the algebra without changing the basic insights.

and, conceivably, the time stream of the publicly provided good $g = \{g^t\}$. Thus $R^t = T^t + \tau\rho A^t(g,T)$.

In this formulation interest payments on government debt are taxable. Therefore government debt evolves according to $D^{t+1} = (1+\rho)D^t + I_g^t - T^t - \tau\rho A^t$. The government's budget constraint can then be written in integrated form as

$\Sigma_{t=0}^{\infty}(R^t - I_g^t)/(1+\rho)^t = D^0$.

where the discount rate is the pre-tax rate of return. Note that even if interest payments on government bonds were tax exempt (so the government could borrow at the after tax rate of return r) the economic opportunity cost of funds is ρ because each dollar of borrowing crowds out a dollar of private investment that yields ρ. Because we are interested in how the project affects the private sector's assets, the discount rate in the government's budget constraint is the pre-tax rate of return.[2]

Now consider a small project that requires a stream of expenditures $\{dI_g^t\}$ and generates a stream of output $\{dg^t\}$. The project is worthwhile if the representative agent is made better off. From the private sector's first order conditions, consumption in each period is a function of the time stream of lump sum taxes and the time stream of the publicly provided good so $c^t(g,T)$. Well-being can therefore be written as $U(c(g,T),g) = V(g,T)$.

Assume (with no loss in generality) that the project is financed by an increase in lump sum taxes in period 0.[3] The project will make the representative agent better off if

$\Sigma_{t=0}^{\infty}(\partial V/\partial g^t)dg^t + (\partial V/\partial T^0)dT^0 > 0$

Dividing through by $\partial V/\partial T^0$ and making use of the envelope theorem, this can be re-written as

$\Sigma_{t=0}^{\infty}\frac{\partial U/\partial g^t}{\partial U/\partial c^0}dg^t - dT^0 > 0$

The project's benefit in period t, denoted by B^t, is equal to $p_g^t dg^t$, where $p_g^t = (\partial U/\partial g^t)/(\partial U/\partial c^t)$ represents the marginal rate of substitution between the publicly provided good and the composite private good in period t. From the first order conditions, $\beta^t(\partial U/\partial c^t)(1+r)^t = \partial U/\partial c^0$. Therefore, the representative agent will be better off if the benefits discounted at the after tax rate of return exceed the required lump sum tax increase in period 0, i.e. if

(1) $\Sigma_{t=0}^{\infty}B^t/(1+r)^t - dT^0 > 0$.

A project requiring a sequence of expenditures $\{dI_g^t\}$ that is financed by a lump sum tax increase dT^0 is fiscally feasible if the present value of the additional tax revenue collected is equal to the present value of the project's expenditure requirements. Thus

(2) $dT^0 + \Sigma_{t=1}^{\infty}dR^t/(1+\rho)^t = \Sigma_{t=0}^{\infty}dI_g^t/(1+\rho)^t$

The second term on the left hand side of (2) captures the combined effects of the project and its financing on the present value of capital income tax revenue.

[2]Since $A^t = K^t + D^t$ the government's budget constraint can be written as $D^{t+1} = (1+r)D^t + I_g^t - T^t - \tau\rho K^t$ which in integrated form becomes $\Sigma_{t=0}^{\infty}(T^t + \tau\rho K^t - I_g^t)/(1+r)^t = D^0$. If we wished to isolate the impact of the project on the capital stock rather than on assets we would use this formulation.

[3]Ricardian equivalence holds in the ILA model, so the *timing* of any lump sum tax increase is irrelevant.

Since $R^t = T^t + \tau\rho A^t$, the change in tax revenue in period $t = 1, ..., \infty$ is

$dR^t = \tau\rho \left[\Sigma_{i=0}^{\infty}(\partial A^t/\partial g^i)dg^i + (\partial A^t/\partial T^0)dT^0\right]$

The first term in this expression represents the "indirect revenue effect" of the project in period t, i.e., the impact of the project on capital income tax revenue in period t. Thus, $IR^t = \tau\rho\Sigma_{i=0}^{\infty}(\partial A^t/\partial g^i)dg^i$. The second term represents the impact on capital income tax revenue in period t of financing the project with a lump sum tax increase in period 0.

Substituting the expression for dR^t the fiscal feasibility constraint (2) can be re-written as

(3) $dT^0 = \left[1 + \tau\rho\Sigma_{t=1}^{\infty}(\partial A^t/\partial T^0)/(1+\rho)^t\right]^{-1} \left[\Sigma_{t=0}^{\infty}dI_g^t/(1+\rho)^t - \Sigma_{t=1}^{\infty}IR^t/(1+\rho)^t\right]$

The first term in square brackets in (3) is the ratio of the increase in lump sum tax revenue to the increase in the present value of total tax revenue. Since the increase in lump sum tax revenue equals the reduction in private sector welfare (all measured in terms of period 0 consumption), it represents the welfare cost per dollar increase in government revenue achieved by a lump sum tax increase. Liu referres to this as the "marginal cost of funds" (MCF) for a lump sum tax. Thus,

(4) $MCF = \left[\Sigma_{t=0}^{\infty}(\partial R^t/\partial T^0)/(1+\rho)^t\right]^{-1} = \left[1 + \Sigma_{t=1}^{\infty}\tau\rho(\partial A^t/\partial T^0)/(1+\rho)^t\right]^{-1}$

Now use (3) to eliminate dT^0 from (1) and we find that the representative agent will be better off with the project provided that

(5) $\Sigma_{t=0}^{\infty}B^t/(1+r)^t - MCF\left[\Sigma_{t=0}^{\infty}dI_g^t/(1+\rho)^t - \Sigma_{t=1}^{\infty}IR^t/(1+\rho)^t\right] > 0$

The present value of the project's benefits discounted at the after tax rate of return r must exceed the present value of the project's expenditure requirements minus any indirect revenue effects discounted at the pre-tax rate of return ρ and mulitiplied by the MCF parameter. This is the MCF criterion proposed by Liu (2003).

According to Liu the SOC criterion for the project to be worthwhile is

$\Sigma_{t=0}^{\infty}(B^t - dI_g^t)/(1+\omega)^t > 0$

where ω is a weighted average of ρ and r. Liu claims that the SOC criterion is flawed because it looks only at a project's direct benefits and costs, ignoring indirect revenue effects, and it fails to take into account that the marginal cost of funds for a lump sum tax exceeds unity when there is a capital income tax in place. He further claims that because the SOC criterion discounts benefits and costs at the same rate, the weighted average discount rate is "project specific" making the SOC criterion almost impossible to implement in practice.

All of these claims are false. They reflect a misunderstanding about the SOC criterion and how to apply it. The appropriate social discount rate reflects what Harberger (1973, Chs 4,5) calls the "social opportunity cost of borrowed funds", here represented by ρ, and it is independent of the project being assessed. The SOC criterion does recognize the possibility of indirect revenue effects, but compared to the MCF criterion indirect revenue effects reflect the **compensated** effect of the project on capital income tax revenue rather than the **uncompensated** effect.[4] Finally, the marginal cost of funds parameter as Liu defines it

[4] The uncompensated effect is the effect of the project on capital income tax revenue holding

does exceed unity for a lump sum tax but it is not necessary to introduce it when applying the SOC criterion unless the uncompensated indirect revenue effect is known and the uncompensated indirect revenue effect is not.[5]

To expand on these points I will first focus (in the next two sub-sections) on the special case of a project that requires an initial expenditure of dI_g^0 and generates a constant stream of output in all subsequent periods so $dg^t = dg$ for $t = 1, ..., \infty$. The analysis is extended to the general case of a project with an arbitrary stream of benefits and costs in sub-section 4. For ease of illustration I assume that the representative agent's pure rate of time preference is equal to the after tax rate of return, and that taxes on labour income are constant over time in the initial equilibrium. Private consumption will then equal permanent (disposable) income so $c^t = c = y = w - T + rA$. In other words, the representative agent consumes the annuity value of wealth.[6] Since $dg^t = dg$ the benefit of the project in each period as measured by the private sector's willingness to pay is $p_g dg = B$. A lump sum tax increase in period 0 of dT^0 will then reduce consumption in period 0 and thereafter by $dc^t = -rdT^0/(1+r)$, assets in period 1 and thereafter will decrease by $dA^t = -dT^0/(1+r)$, and capital income tax revenue in period 1 and thereafter will decrease by $dR^t = -\tau\rho dT^0/(1+r)$. The marginal cost of funds parameter for a lump sum tax as defined by Liu simplifies to $MCF = \rho(1+r)/r(1+\rho)$.[7]

How a project impacts distorted markets is an important factor in project assessment. Therefore, in the next two sub-sections I look at two characterizations of the project's benefits that have drawn the most attention in the literature: project benefits being fully consumed, and project benefits being treated as income.

private sector income fixed. It is the **Marshallian** uncompensated effect. Hatta (1977) shows that the compensated effect of the project is equal to the uncompensated **Bailey** effect divided by the Hatta coefficient, a parameter representing the shadow value of government revenue. The uncompensated Bailey effect is the effect of the project on capital income tax revenue with private sector income adjusted to balance the government's budget. Thus the uncompensated Bailey effect is a scalar multiple of the compensated effect. For further details see Jones (2005).

[5] The conventional (Harberger) measure of the MCF is one plus the marginal excess burden, where the marginal excess burden is computed by marginally raising the tax and returning the revenue to taxpayers as a lump sum transfer. Thus the conventional MCF for a lump sum tax is unity. Liu's MCF parameter for a lump sum tax exceeds unity because the government retains the revenue and the impact of a lump sum tax increase on the government's budget differs from its impact on welfare (present value of private consumption) because private consumption is discounted at the after tax rate whereas tax revenue is discounted at the pre-tax rate. Alternative concepts of the MCF are well explained by Jones (2005).

[6] The results can be easily generalized to situations where the after tax rate of return differs from the pure rate of time preference. Thus if r differs from $\delta = 1 - 1/\beta$ then the growth rate of consumption $g(c^t)$ is equal to $(r-\delta)/\eta$ where η is the (constant) elasticity of the marginal utility of consumption.

[7] If the representative agent's pure rate of time preference differs from the after tax rate of return then $r = \delta + g\eta$. In this case consumption grows at rate g and the MCF parameter for a lump sum tax becomes $(\rho - g)(1+r)/[(r-g)(1+\rho)]$.

2.1 Project Benefits Fully Consumed

A project may or may not affect private sector behaviour, even though the private sector has a willingness to pay for the project's output. Suppose the preference function for the representative agent takes the additively separable form

$\Sigma_{t=0}^{\infty}\beta^t[U(c^t) + f(g^t)]$

where $U(.)$ and $f(.)$ are monotonic increasing and concave. Then an increase in g^t holding $\{T^t\}$ fixed will leave the time path of private consumption unaffected, i.e. $\partial c^i/\partial g^t = 0$, for all i. The project's benefits will be "fully consumed" in the period in which they are produced so the project leaves no trace in terms of private consumption, a property known as "uncompensated independence".[8] A project that has no impact on private consumption will have no impact on saving, and therefore no impact on assets or capital income tax revenue. In other words, the uncompensated Marshallian indirect revenue effect of the project will be zero. Setting $IR^t = 0$ in (5), the MCF criterion simplifies to

$\Sigma_{t=0}^{\infty}B^t/(1+r)^t - MCF[\Sigma_{t=0}^{\infty}dI_g^t/(1+\rho)^t] > 0$

For a project requiring an initial expenditure of dI_g^0 and producing a perpetual stream of benefits worth B beginning in period 1, with the benefits being fully consumed, the MCF criterion simplifies to

(6) $B/r - MCF.dI_g^0 > 0.$

How should the SOC criterion be applied in this situation? Liu (2003) claims that the SOC criterion looks only at a project's direct benefits and costs, ignoring indirect revenue effects, and it fails to take into account that the MCF parameter exceeds unity for a lump sum tax. Thus, if indirect revenue effects are zero as in (6) the project will be worthwhile provided that $B/\omega - dI_g^0 > 0$, where $\omega = r.MCF = \rho(1+r)/(1+\rho)$ is the appropriate discount rate- a weighted average of ρ and r. According to Liu (2003) the SOC criterion results in "project specific" discount rates.[9] However, the SOC criterion proposed by Harberger (1973, Chs 4,5) requires that benefits and costs (including any indirect revenue effects) be discounted at the "social opportunity cost of borrowed funds".[10] This is the rate of return foregone when the government, at pre-existing tax rates,

[8] Wildasin (1984) emphasizes the distinction between compensated and uncompensated independence in deriving criteria for the optimal supply of a public good in a static model. See also Browning (1987). Note that in a static model we need only weak separability between the private consumption vector $c = c^1, ..., c^n$ and the publicly provided good g to ensure that an increase in g has no behavioural impact, *i.e.* no impact on tax revenue. In an intertemporal model with a capital income tax we need strong separability between $\{c^t\}$ and $\{g^t\}$ to avoid any behavioural impact.

[9] For example, a project requiring an initial expenditure of dI^0 and generating (fully consumed) benefits worth B beginning in period 2 instead of period 1 is worthwhile according to the MCF criterion if $B/r(1+r) - MCF.dI^0 > 0$. Since $MCF = \rho(1+r)/r(1+\rho)$ it is easy to verify that the discount rate that makes this project just worthwhile is the value of ω such that $B/\omega(1+\omega) - dI^0 = 0$. This is a different value for ω than for a project whose benefits begin in period 1.

[10] The original term was the social opportunity cost of *public* funds, but it is now referred to as the social (economic) cost of *borrowed* funds to avoid confusing it with the conventional (Harberger) marginal cost of funds (MCF).

induces the private sector voluntarily to relinquish funds that would otherwise finance private consumption and investment. It is a weighted average of the pre-tax and post tax rates of return, where the weights reflect the proportions of funding that displace private investment and consumption respectively when the government borrows to finance the project.[11] If the pre-tax rate of return ρ is exogenous the SOC rate will equal the pre-tax rate of return because each dollar of funding displaces a dollar of private investment.

The benchmark for measuring indirect revenue effects using the SOC criterion is a project whose benefits are equivalent to income. A project with an output stream $\{g^t\}$ is equivalent to income if $\partial c^i/\partial g^t = p_g^t \partial c^i/\partial y^t$ for all i, where y^t is income in period t and p_g^t is the marginal willingness to pay for a unit of the project's output in period t. In other words, providing the project and charging individuals their marginal willingness to pay for it leaves consumption unchanged.[12] But according to the Slutsky equation, $\partial c^i/\partial g^t = (\partial c^i/\partial g^t)_u + p_g^t \partial c^i/\partial y^t$, so the SOC criterion takes as its benchmark a project for which the compensated effect on consumption in all periods is zero, i.e. $(\partial c^i/\partial g^t)_u = 0$, for all i.

For a project whose benefits are fully consumed $\partial c^i/\partial g^t = 0$ for all i, in which case $(\partial c^i/\partial g^t)_u = -p_g^t \partial c^i/\partial y^t < 0$; the compensated effect of the project on consumption in all periods is negative, so the project decreases consumption in all periods relative to the benchmark.

In applying the SOC criterion the indirect revenue effect in each period reflects the *compensated* effect of the project on capital income tax revenue in that period. Since $IR^t = \tau\rho\Sigma_{i=0}^{\infty}(\partial A^t/\partial g^i)dg^i$, then $IR_c^t = IR^t - p_g^i(\partial A^t/\partial y^i)dg^i$. But $p_g^i dg^i = B^i$ and $\partial A^t/\partial y^i = -\partial A^t/\partial T^i$, so

(7) $IR_c^t = IR^t + \tau\rho\Sigma_{i=0}^{\infty}B^i(\partial A^t/\partial T^i)$.

For a project whose uncompensated effect on consumption (and therefore capital income tax revenue) is zero, $IR^t = 0$ for all t, so the compensated effect on capital income tax revenue in period t is simply the effect on capital income tax revenue in that period of a sequence of lump sum tax increases equal to the project's benefits.

In the case of a project that provides a constant stream of benefits worth B in periods $t = 1, 2, ..., \infty$ that the private sector fully consumes, the indirect revenue effects reflect the effect on capital income tax revenue of an increase in

[11] Sjaastad and Wisecarver (1977, p 533) emphasize that Harberger's SOC rate refers only to the raising of funds, while acknowledging that how the funds are spent can affect private sector decisions. Sandmo-Dreze (1971) derive Harberger's SOC rate as the marginal rate of return on public investment at the second best optimum under the pre-existing capital income tax. They assume that public investment is a perfect substitute for private investment. Burgess (1988) shows that Harberger's SOC rate applies to projects that are complements or substitutes for private investment provided that the project's effect on capital income tax revenue through the shift in investment demand is added to, or subtracted from, the benefits.

[12] Harberger (1973, pp. 47-48) explicitly states that whenever the project induces a shift in demand or supply in any market that is distorted by a tax, the magnitude of the shift should be multiplied by the tax wedge and the resulting indirect revenue effects should be added to (or subtracted from) the benefits, which are discounted at the SOC rate. What is not made explicit is that it is the *compensated* effect of the project on distorted markets that is relevant.

lump sum taxes of $dT^t = B$ in periods $t = 1, 2, ...\infty$. Consumption will fall by $dc = -B/(1+r)$ in all periods and assets will increase by $B/(1+r)$ in periods $t = 1, 2, ...\infty$, so capital income tax revenue will increase by $\tau\rho B/(1+r)$ in periods $t = 1, 2, ...\infty$. Therefore, insofar as the SOC criterion is concerned, the indirect revenue effect of the project in period $t = 1, 2, ...\infty$ is $IR_c^t = \tau\rho B/(1+r)$.[13]

Adding the indirect revenue effects to the benefits and discounting at the SOC rate ρ, the project is worthwhile according to the SOC criterion if

(8) $B/\rho + IR_c/\rho - dI_g^0 > 0$

To confirm that the SOC criterion is equivalent to the MCF criterion, substitute for $IR_c = \tau\rho B/(1+r)$ to obtain the condition

$B/\rho + \tau\rho B/(1+r)\rho - dI_g^0 > 0$

Next recall that the MCF parameter is equal to $\rho(1+r)/r(1+\rho)$ so this can be re-written as

$B/r - MCF.dI_g^0 > 0$

which is the MCF criterion expressed in (6).

So the SOC criterion is perfectly valid for projects whose benefits are fully consumed; benefits and costs should be discounted at the SOC rate, but the compensated effect of the project on capital income tax revenue must be added to the benefits.

2.2 Project Benefits Treated as Income

The assumption that the project has no impact on private sector behaviour is very special. Indeed, it has been argued that if a person who experiences a change in the quantity of a publicly provided good takes no action whatsoever in response the publicly provided good cannot be a significant source of value for the person.[14] More plausible is the notion that the publicly provided good g^t and contemporaneous private consumption c^t are substitutes rather than independents or complements. An increase in g^t then lowers the marginal utility of private consumption in period t leaving the marginal utility of private consumption in all other periods unchanged. The individual responds by smoothing consumption, i.e. reducing private consumption in period t and increasing private consumption in all other periods.[15]

Suppose the preference function for the representative agent takes the form

$\Sigma_{t=0}^{\infty}\beta^t U(c^t, q(e^t, g^t))$

where $U(.)$ and $q(.)$ are monotonic increasing and concave functions of their arguments. In this formulation private expenditure in period t consists of two components: ordinary private consumption c^t, and "averting expenditure" e^t that is motivated solely by the desire to mitigate the adverse effects of the

[13] Since $IR_c^t = \tau\rho\Sigma_{i=0}^{\infty}B^i(\partial A^t/\partial T^i)$, when $B^i = B$ for $i = 1, 2, ...\infty$, then $IR_c^t = \tau\rho B\Sigma_{i=1}^{\infty}\partial A^t/\partial T^i$. But $\partial A^t/\partial T^i = (\partial A^t/\partial T^0)(1+r)^{-i}$ and $\partial A^t/\partial T^0 = r/(1+r)$ for $t = 1, 2, ...\infty$. Therefore $IR_c^t = \tau\rho B/(1+r)$.

[14] See e.g. Larson (1993). Note that the strong separability restriction has the implausible implication that neither the composite private good nor the publicly provided good is essential.

[15] If g^t and c^t were complements, the individual would respond by increasing consumption in period t and reducing consumption in all other periods which seems counter-intuitive for an individual striving to smooth consumption over her lifetime.

limited supply of the publicly provided good g^t. One can imagine the publicly provided good g^t serving as an input along with expenditures on a private good e^t in the "household production" of q^t. The formulation therefore allows g^t and e^t to be imperfect substitutes in the production of q^t.[16]

An increase in g^t holding $\{T^t\}$ fixed will increase c^i in all periods including period t, increase e^i in all periods except period t, but reduce e^t and "private spending" in period t (defined as $c^t + e^t$). If sufficient income is then taken away to keep utility fixed, c^i and e^i will be unchanged for all i and only e^t will decrease, with the decrease in e^t being equal to the reduction in income required to keep utility fixed.[17] In other words, $(\partial c^i/\partial g^t)_u = \partial c^i/\partial g^t - p_g^t(\partial c^i/\partial y^t) = 0$, for all i. The private sector will respond to an increase in g^t in the same way as it would to an increase in a lump sum transfer equal to the public's willingness to pay for g^t. Since $(\partial c^i/\partial g^t)_u = 0$ for all i, the uncompensated response is positive. Specifically, $\partial c^i/\partial g^t = p_g^t(\partial c^i/\partial y^t) > 0$ for all i.

If the compensated effect of the project on consumption in all periods is zero, the compensated effect on assets (and therefore capital income tax revenue) will also be zero in all periods. We have already shown that $IR_c^t = IR^t + \tau\rho\Sigma_{i=0}^{\infty}B^i(\partial A^t/\partial T^i)$, so if $IR_c^t = 0$ then $IR^t = -\tau\rho\Sigma_{i=0}^{\infty}B^i(\partial A^t/\partial T^i)$. The indirect revenue effect in each period is the effect on capital income tax revenue in that period of a sequence of lump sum transfers equal to the project's benefits.

In the case of a project that provides a constant stream of benefits worth B in periods $t = 1, 2, ...\infty$ that the private sector treats as income, the indirect revenue effects in applying the MCF criterion reflect the impact on capital income tax revenue of a sequence of lump sum transfers equal to B in periods $t = 1, 2, ...\infty$. Consumption will increase by $dc = B/(1 + r)$ in all periods and assets will decrease by $B/(1 + r)$ in periods $t = 1, 2, ...\infty$, so capital income tax revenue will decrease by $\tau\rho B/(1 + r)$ in periods $t = 1, 2, ...\infty$. Formally, since $IR^t = -\tau\rho\Sigma_{i=1}^{\infty}B^i(\partial A^t/\partial T^i)$, if $B^i = B$ then $IR^t = -\tau\rho B\Sigma_{i=1}^{\infty}(\partial A^t/\partial T^i) = -\tau\rho B\Sigma_{i=1}^{\infty}(\partial A^t/\partial T^0)(1 + r)^{-i} = -\tau\rho B/(1 + r)$ for $t = 1, 2, ...\infty$.

The MCF criterion requires that indirect revenue effects be subracted from the project's costs, discounted at the pre-tax rate ρ and multiplied by the MCF parameter. Therefore the project is worthwhile according to the MCF criterion if

(9) $B/r - MCF\left\{dI_g^0 + \tau B/(1 + r)\right\} > 0$

However, (9) can be simplified by substituting for $MCF = \rho(1+r)/r(1+\rho)$ and re-arranging terms. As a result, the project is worthwhile provided that

(10) $B/\rho - dI_g^0 > 0$

which is the SOC criterion proposed by Harberger (1973) and Sandmo-Dreze (1971). Simply put, the project is worth doing as long as its benefits discounted at the SOC rate exceeds its costs. When the project's benefits are equivalent to income there are no "indirect revenue effects" to take into account.

[16] The special case of g^t being a perfect substitute for some private good e^t is included in this specification, but it is unnecessarily restrictive. See Liu et al (2005).

[17] This is an extension to an intertemporal context of the model of defensive spending formulated by Courant and Porter (1981) and Bartik (1988).

2.3 Choice of Benchmark

We have found that there is no conflict between the SOC criterion and the MCF criterion. The criteria differ in terms of the benchmark that is used to measure indirect revenue effects. While the choice of benchmark is a matter of taste, the reality is that indirect revenue effects are difficult to measure (and typically ignored in project evaluation). Therefore, it is best to choose a criterion where ignoring indirect revenue effects is least objectionable. On these grounds, there are at least four situations where the SOC criterion would seem to be the preferred choice.

First, if a project yields benefits that are appropriable via user fees or normal market transactions, (i.e. benefits that the private sector can provide) the appropriate discount rate is the SOC rate. Projects like electricity generation or water and wastewater treatment services would fall into this category.

Second, for projects that yield benefits that the private sector appropriates as a component of full income the appropriate discount rate is the SOC rate. Education and training programs that improve labour skills thereby raising real wages, or infrastructure projects whose benefits are reflected in higher land values and rents would fall into this category.

Third, the SOC rate is appropriate for projects that provide services that are a perfect substitute for some privately produced good such as "free" school lunch programs, or public health care that reduces the demand for private health care.

Fourth, if individuals purchase some market goods to mitigate the adverse effects of environmental bads, the benefits of an increment in government spending to improve environmental quality can be measured by the reduction in private spending on mitigation. Examples include the purchasing water filters or air purifiers to defend against poor air or water quality.

In each of these situations the project can be evaluated simply by determining whether the benefits (measured by willingness to pay) exceed the costs discounted at a rate that reflects the economic opportunity cost of borrowed funds. While it is true that i) the consumption rate of interest correctly converts the private sector's willingness to pay into its present value, ii) the MCF parameter for a lump sum tax (as defined by Liu) exceeds unity if there is a pre-existing capital income tax, and iii) the project has a direct effect on capital income tax revenue, (i.e. there are indirect revenue effects as Liu defines this term), these facts in combination are *irrelevant* in evaluating a project whose benefits are equivalent to income.

2.4 The General Result

It is, of course, conceivable that a project's benefits are neither fully consumed nor equivalent to income. In this case there will be indirect revenue effects to incorporate no matter which criterion is used. The compensated and uncompensated indirect revenue effects in period t of a project with benefits $\{B^i\}_{i=0}^{\infty}$ are related to each other in equation (7). The indirect revenue effect of the

project is the discounted sum of the effects on capital income tax revenue in each period, using the pre-tax rate of return as the discount rate. Therefore for a project with benefits $\{B^i\}_{i=0}^{\infty}$the compensated and uncompensated indirect revenue effects are related by

(11) $\Sigma_{t=1}^{\infty} IR_c^t/(1+\rho)^t = \Sigma_{t=1}^{\infty} IR^t/(1+\rho)^t + \tau\rho\Sigma_{i=0}^{\infty} B^i \Sigma_{t=1}^{\infty}(\partial A^t/\partial T^i)/(1+\rho)^t$

Liu defines the marginal cost of funds parameter for a particular tax as the cost in terms of current consumption per dollar increase in government revenue from the tax increase. For a lump sum tax increase in period 0 the MCF parameter is given in (4). The marginal cost of funds parameter is independent of the *timing* of the tax increase. Thus for a lump sum tax increase announced in period 0 that comes into effect in period i

(12) $MCF_{T_i} = (1+r)^{-i}/[(1+\rho)^{-i} + \Sigma_{t=1}^{\infty}\tau\rho(\partial A^t/\partial T^i)/(1+\rho)^t] = MCF_{T_0}$

Rearranging terms and dropping the subscript on the MCF parameter we have

(13) $\Sigma_{t=1}^{\infty}\tau\rho(\partial A^t/\partial T^i)/(1+\rho)^t = (1+r)^{-i}/MCF - (1+\rho)^{-i}$

In words, a dollar increase in lump sum taxes in period i will reduce welfare (the present value of private consumption) by $(1+r)^{-i}$, which will increase the present value of government revenue by $(1+r)^{-i}/MCF$ and increase the present value of lump sum tax revenue by $(1+\rho)^{-i}$. The difference is the impact of a dollar increase in lump sum taxes in period i on the present value of capital income tax revenue.

For a project with benefits $\{B^i\}_{i=0}^{\infty}$the compensated indirect revenue effect can now be related to the uncompensated Marshallian indirect revenue effect by

(14) $\Sigma_{t=1}^{\infty} IR_c^t/(1+\rho)^t = \Sigma_{t=1}^{\infty} IR^t/(1+\rho)^t + \Sigma_{i=0}^{\infty} B^i[(1+r)^{-i}/MCF - (1+\rho)^{-i}]$

Recall that the inverse of the MCF parameter represents the increase in government revenue per dollar reduction in private sector welfare. Therefore, the term $\Sigma_{i=0}^{\infty} B^i(1+r)^{-i}/MCF$ measures the increase in the present value of government revenue from a sequence of lump sum tax increases equal to the public's willingness to pay for the project's benefits, and the term $\Sigma_{i=0}^{\infty} B^i(1+\rho)^{-i}$ measures the corresponding increase in the present value of lump sum tax revenue. The difference is therefore the impact on capital income tax revenue of a sequence of lump sum tax increases equal to the project's benefits.

According to the SOC criterion the project will be worthwhile if the benefits plus the indirect revenue effects (defined as the compensated effect of the project on capital income tax revenue) exceed the costs, where benefits, costs and indirect revenue effects are discounted at the economic opportunity cost of capital (equal to the pre-tax rate of return if ρ is exogenous).

Thus the project is worthwhile if

(15) $\Sigma_{t=0}^{\infty}[B^t + IR_c^t - dI_g^t]/(1+\rho)^t > 0$

The compensated indirect revenue effect can be related to the uncompensated indirect revenue effect using (14). When the substitution is made the criterion becomes

(16) $\Sigma_{t=0}^{\infty}[B^t + IR^t - dI_g^t]/(1+\rho)^t + \Sigma_{t=0}^{\infty} B^t[(1+r)^{-t}/MCF - (1+\rho)^{-t}] > 0$

which simplifies to

(17) $\Sigma_{t=0}^{\infty} B^t/(1+r)^t + MCF.\Sigma_{t=0}^{\infty}[IR^t - dI_g^t]/(1+\rho)^t > 0$

which is the MCF criterion.

Thus if the compensated indirect revenue effect is zero the uncompensated (Marshallian) indirect revenue effect can be readily calculated from estimates of the project's benefits and the MCF parameter. Specifically, if $\Sigma_{t=1}^{\infty} IR_c^t/(1+\rho)^t = 0$ then $\Sigma_{t=1}^{\infty} IR^t/(1+\rho)^t = \Sigma_{t=0}^{\infty} B^t[(1+\rho)^{-t} - (1+r)^{-t}/MCF]$. However, rather than calculate indirect revenue effects in this situation it is easier to use the standard SOC criterion (with no indirect revenue effects). Conversely, if the uncompensated indirect revenue effect of the project is zero the compensated indirect revenue effect is readily calculated from estimates of the project's benefits and the MCF parameter. Specifically, if $\Sigma_{t=1}^{\infty} IR^t/(1+\rho)^t = 0$ then $\Sigma_{t=1}^{\infty} IR_c^t/(1+\rho)^t = \Sigma_{t=0}^{\infty} B^t[(1+r)^{-t}/MCF - (1+\rho)^{-t}]$. However, in this situation it is easier to use the MCF criterion (with no indirect revenue effects).

It can be argued that an advantage of the MCF criterion is that it separates the project specific indirect revenue effects from the financing specific MCF parameter, whereas the SOC criterion combines both into the compensated indirect revenue effect, so the same project will have a different indirect revenue effect under the SOC criterion depending upon how it is financed. But the SOC criterion applies a level playing field to all projects by assuming the same source of funding for all, namely the capital market. Project assessment is thereby divorsed from issues of tax reform, thereby avoiding situations where a project is judged worthwhile because its method of financing is a move toward a more efficient tax regime. It is true that the SOC criterion in (15) is valid only if lump sum taxes are feasible, whereas the MCF criterion in (17) applies for any choice of tax instrument. If distortionary taxes must be used the SOC criterion requires that the project's costs plus indirect revenue effects be multiplied by a parameter equal to one plus the marginal excess burden of the distortionary tax. The appropriate discount rate is still the economic opportunity cost of borrowed funds.

3 Shadow Price Algorithm

Now let us look at the social rate of time preference approach (STP) proposed by Marglin (1964), Feldstein (1972), Bradford (1975) and Lind (1982). These authors claim that the SOC criterion commits an "aggregation error" (for multi-period projects) by attempting to combine two distinct prices- the price of future consumption in terms of present consumption, and the price of investment in terms of contemporaneous consumption- into one discount rate. The appropriate procedure, they believe, is to convert benefits and costs into "consumption equivalents" by shadow pricing all investment displaced or induced and discounting at the social rate of time preference (here represented by the after tax rate of return).[18] For a project requiring an initial expenditure of dI_g^0 and pro-

[18] For a recent defence of this position see Dasgupta (2008, p 156). Proponents of the STP approach allow the social rate of time preference to differ from the after tax rate of return

ducing a constant stream of output dg beginning in period 1 worth $B(dg)$, with these benefits being "fully consumed", the "shadow price algorithm" judges the project to be worthwhile provided that

(18) $B/r - [(1-\gamma) + \gamma SPC)]dI_g^0 > 0.$

Here γ is the proportion of project expenditure that displaces private investment (so $(1-\gamma)$ is the proportion that displaces consumption), and SPC is the "shadow price of capital", which is the present value of the stream of consumption foregone when a dollar of private investment is displaced. If private investment earns a pre-tax rate of return of ρ and the representative agent consumes the annuity value of wealth, a dollar of private investment is worth ρ/r dollars of contemporaneous consumption.

Clearly, the shadow price algorithm will be equivalent to the MCF criterion given in (6) only if the MCF parameter happens to equal $(1-\gamma)+\gamma SPC$. Most practitioners have assumed that γ represents the marginal propensity to save.[19] Thus, if the government raises lump sum taxes to finance the project $1-\gamma$ is the proportion of the lump sum tax increase that comes from consumption. If the representative agent consumes the annuity value of wealth, the proportion that comes from consumption will be $r/(1+r)$.[20] It is then inferred that the proportion of **project expenditure** that comes from consumption is $r/(1+r)$ and therefore the proportion that comes from investment is $1/(1+r)$.

But this is incorrect. If the cost of the project is dI_g^0 the present value of the required increase in lump sum taxes is $dT^0 = MCF.dI_g^0$, so the amount drawn from consumption in period 0 is $(1-\gamma)dT^0 = rdT^0/(1+r) = MCF.dI_g^0.r/(1+r) = \rho dI_g^0/(1+\rho)$, and therefore the amount drawn from investment is $dI_g^0/(1+\rho)$. Using these proportions it is easy to see that $(1-\gamma)+\gamma SPC = MCF$. Notice that even if the government were to set $dT^0 = dI_g^0$, as long as the private sector is fully informed about the future lump sum tax increase required to finance the project it will respond so as to minimize the present value of consumption foregone.[21]

What if the project requires multi-period expenditures? Practitioners have assumed that as long as γ and SPC are constants each dollar of project expenditure will have the same consumption equivalent, namely $1-\gamma+\gamma SPC$, no matter when that dollar is spent. But this depicts the project as a sequence of expenditures that the private sector fails to anticipate, even though these expenditures are known by the planner. If the planner discloses the full cost of the project (by specifying the time sequence of required lump sum tax in-

(consumption rate of interest), but we set aside these issues in this paper.

[19] See e.g. Bradford (1975), Mendelshon (1981) and Lind (1982).

[20] If the private sector consumes the annuity value of wealth then $c(1+r)/r = \Sigma_{t=0}^{\infty}(w - T^t)/(1+r)^t + A^0$. An increase in lump sum taxes of dT^0 in period 0 will reduce consumption in all periods by $dc = -rdT^0/(1+r)$.

[21] To maintain a balanced budget the government could raise lump sum taxes by $dT^0 = dI^0$ in period 0, but it would subsequently have to raise lump sum taxes further to offset the loss in capital income tax revenue. The **present value** of the lump sum tax increase necessary to finance the project is therefore $dT^0 = MCF.dI^0$. The rational agent with perfect foresight will anticipate the tax increase that is required to finance the project and adjust her consumption and saving accordingly.

creases) the private sector will respond so as to minimize the present value of consumption foregone.

By treating the private sector as myopic with respect to the project's costs the shadow price algorithm will overstate the present value of the consumption foregone in financing any project. It follows that for projects whose benefits are fully consumed the shadow price algorithm will be overly stringent. If the private sector is fully informed about the project's costs a dollar of project expenditure will have a different "consumption equivalent" depending upon when that dollar is spent; project expenditure at time t must be converted into its consumption equivalent by multiplying by $MCF[(1+r)/(1+\rho)]^t$ and then discounting at the after tax rate of return r.[22]

What about projects whose benefits are treated as income? If the private sector is fully informed it will treat the project's benefits as equivalent to income and respond by smoothing consumption. Since Ricardian equivalence holds it is immaterial whether the government finances the project by increasing lump sum taxes in period 0 or by borrowing, thereby deferring the lump sum tax increase to the future. A lump sum tax increase in period 0 to finance expenditures on a project that promises to deliver a stream of lump sum transfers in subsequent periods will not reduce consumption in period 0 if the project is worth undertaking; the lump sum tax increase will reduce saving and investment in period 0 dollar for dollar. For a project that requires an initial expenditure of dI_g^0 and produces a perpetual stream of benefits worth B that the private sector treats as income the shadow price algorithm becomes

$$B/r - SPC.dI_g^0 > 0$$

where $SPC = \rho/r$ if the private sector consumes the annuity value of wealth.[23] Clearly the STP criterion is equivalent to the SOC criterion, a point made by Sjaastad and Wisecarver (1977). On the other hand, the MCF criterion is given in equation (6), which amounts to the same thing.

This equivalence can be extended to more complicated projects, but it requires care in converting project costs into their consumption equivalents. For example, if the project's benefits B do not begin until period $T > 1$ there will be interim financing costs to take into account. Resources must be reallocated from investment to consumption in periods $t = 1, 2, ..., T-1$ to maintain the pre-project consumption plan, and each dollar reallocated increases the present value of consumption displaced by $\rho/r - 1$ dollars. Consequently, the consumption equivalent value of the investment displaced in financing the project is $(\rho/r)[(1+\rho)/(1+r)]^{T-1}dI_g^0$. The project is therefore worthwhile according to the shadow price algorithm if

$$B/r(1+r)^{T-1} - (\rho/r)[(1+\rho)/(1+r)]^{T-1}dI_g^0 > 0$$

But this is equivalent to the SOC criterion

$$B/(1+\rho)^{T-1}\rho - dI_g^0 > 0$$

[22] Liu (2003) makes this point, but he does not draw the implication that the STP criterion as conventionally applied will be overly stringent for projects whose benefits are fully consumed.

[23] Even though the benefits are treated as income (and therefore investible) there is no shadow pricing of benefits because a fully informed individual who consumes the annuity value of wealth will fully consume the benefits B each period.

A further example is a project that costs dI_g^0 but produces all its benefits B^1 in period 1. If the benefits are anticipated and treated as income, financing the project will displace private investment dollar for dollar. However, the present value of consumption foregone when a dollar of investment is displaced is just the value of consumption foregone for one period, so $SPC = (1+\rho)/(1+r)$. The shadow price algorithm applied to this project is then

$B1/(1+r) - [(1+\rho)/(1+r)]dI_g^0 > 0$

which is equivalent to the SOC criterion.[24]

In sum, if the private sector is fully informed about the project's benefits and costs the STP criterion and the SOC criterion are equivalent. The claim that the SOC criterion commits an aggregation error by combining two distinct prices - the shadow price of investment and the consumption rate of interest- into one discount rate, is false. The social (economic) opportunity cost of borrowed funds reflects both the social rate of return on incremental saving and the social rate of return on displaced investment. There is no need to separate these two prices when performing social cost benefit analysis. Whether the benefits are fully consumed or treated as income, the STP criterion encounters problems of implementation if the private sector is fully informed about the project's benefits and costs.

4 Concluding Remarks

Liu (2003) claims that the SOC criterion suffers from "severe implementation problems". Specifically, there is no general formula for the proportions of funding that are drawn from consumption and investment; the proportions depend upon the project, making the discount rate "project specific". Liu is interpreting the SOC as the internal rate of return on a project (specified by its direct benefits and costs, excluding any indirect revenue effects) that is just worthwhile. However, if we follow Harberger (1973) and define the SOC as the "social opportunity cost of borrowed funds", i.e. the rate of return foregone when the government borrows to finance a project, the SOC rate will be unique and common to all projects. If there are indirect revenue effects they should be added to or subtracted from the project's benefits, not incorporated by adjusting the discount rate. In Liu's MCF criterion the appropriate discount rate requires no weights; what is needed are estimates of the pre-tax and post-tax rates of return and the MCF parameter, all of which are project independent. However, Liu's MCF criterion is valid only in circumstances where the pre-tax rate of return is exogenous, because only then will the pre-tax rate of return represent the economic opportunity cost of government spending. Under these circumstances

[24] Alternatively, the gross benefits B^1 must be separated into their genuine income component $B^1 - dI_g^0$, for consumption and the depreciation component dI_g^0 for investment. The project is worthwhile according to the shadow price algorithm if $[B^1 - dI_g^0 + (\rho/r)dI_g^0]/(1+r) - dI_g^0(\rho/r) > 0$, which simplifies to the criterion in the text.

the SOC rate will equal the pre-tax rate of return so, contrary to Liu, no weights are required to implement the SOC criterion.

We have found that when the pre-tax rate of return is exogenous the SOC criterion and the MCF criterion both correctly identify all worthwhile projects. However, each may have an implementation advantage in particular circumstances. If benefits are treated as income the SOC criterion has an implementation advantage because no indirect revenue effects need to be taken into account. If benefits are fully consumed Liu's MCF criterion has an implementation advantage for the same reason. If benefits are neither fully consumed nor treated as income there will be indirect revenue effects to take into account using either criterion, and since there is a well defined relationship between the indirect revenue effects that apply to each criterion they will be equally difficult to measure in practical applications.

The STP criterion proposed by Marglin, Feldstein, Bradford and Lind can be reconciled with the SOC and MCF criteria if the private sector is fully informed about the project's benefits and costs, but it faces serious problems of implementation because a dollar of project expenditure will have a different "consumption equivalent" depending upon when that dollar is spent.

Throughout the pre-tax rate of return is assumed to be exogenous. The "weighted average" discount rate is then simply the pre-tax rate of return. In models where the pre-tax rate of return is endogenous, the economic opportunity cost of borrowed funds will be a weighted average of the pre-tax and after-tax rates of return, with the weights representing the proportions of funding drawn from investment and consumption respectively when the government induces the private sector voluntarily to relinquish funds at pre-existing tax rates. In the context of a model with a finite number of periods Diamond (1968) and Dreze (1974) argued that the weights used in the SOC formula depend upon the duration of the project. But these authors interpreted the discount rate as the internal rate of return on a project specified by its benefits and costs excluding any indirect revenue effects that is just worth doing. If the SOC is defined as the economic opportunity cost of borrowed funds it will be unique and common to all projects provided that the tax distortions are given and the economy is in equilibrium with the distortions in place. Thus what is needed for project evaluation are estimates of benefits measured by willingness to pay, estimates of direct project expenditures, and estimates of indirect revenue effects whenever the project's compensated impact on distorted markets differs from zero. The appropriate discount rate is the social (economic) opportunity cost of capital, and it is independent of the project.

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